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COMMENT

Comment on the transcendental method in the theory of neutron slowing down

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Abstract. The purpose of the paper, to which I give a comment, was to obtain an exact analytical closed-form solution of the linear one-dimensional integral equation of the neutron slowing down with the energy-dependent cross section in an infinite homogenous medium. The transcendental method used in that paper is based on the solvability of the transcendental equation. It is shown in this comment that the function which is recommended as the solution of the transcendental equation does not, in fact, satisfy it.

1. Exact solution of the transcendental equation

The exact solution of the linear integral one-dimensional equation of neutron slowing down is based on two premises. The first premise is that the total collision density $F(u)$ has the form

$$F_\delta(u) = (1 - \Psi_u(0)) f_0(u) \exp(-\varepsilon_u S(u)) + \delta(u) \tag{1}$$

(see equation (47) in Perovich (1992)). The second premise is that the transcendental equation of the form

$$Z_1(u) = B_1(u) \exp(B_0(u) Z_1(u)) \tag{2}$$

has an exact solution (equation (61) in Perovich (1992)). The solution (72) (Perovich 1992) of equation (61) was obtained using the incorrect assumption that the functions (68) and (69) can be made to coincide.

Let us see some properties of the function

$$\Phi(y) = \Phi_0 \sum_{n=0}^{[y/B_0(u)]} (-1)^n B_1^n(u)(y - nB_0(u))^n/n!$$

where $[y/B_0(u)]$ denotes the greatest integer less than or equal to $y/B_0(u)$. For $1 < [y/B_0(u)] < 2$

$$\Phi(y) = \Phi_0 (1 - B_1(u)(y - B_0(u))) \qquad \text{(polynomial of order 1)}$$

for $2 < [y/B_0(u)] < 3$

$$\Phi(y) = \Phi_0 \left(1 - B_1(u)(y - B_0(u)) + \frac{B_1(u)^2}{2}(y - 2B_0(u))^2\right)$$

(polynomial of order 2)

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and for $m < [y/B_0(u)] < m+1$

$$\Phi(y) = \Phi_0 \sum_{n=0}^{m} (-1)^n B_1^n(u)(y - nB_0(u))^n/n! \qquad \text{(polynomial of order m).}$$

In every single interval $(m, m+1)$ $(m = 0, 1, \ldots)$ the function $\Phi(y)$ behaves as a polynomial of order m. Perovich states that there exists some undefined y_0 such that for all $y > y_0$ we could establish the equality

$$\Phi_{p0} \exp(-Z_1(u)y) = \Phi_0 \sum_{n=0}^{[y/B_0(u)]} (-1)^n B_1^n(u)(y - nB_0(u))^n/n!. \tag{3}$$

It is easy to prove that this is not true. If these functions are equal, then their derivatives must also be equal. If we differentiate equation (3) by y at some point $y_1 > y_0$ $[y_1/B_0(u)] + 1$ times, then, as a result, we will obtain zero on the right-hand side (note that the upper limit in the sum is then fixed) and on the left-hand side we will obtain

$$\Phi_{p0}(-Z_1(u))^{[y_1/B_0(u)]} \exp(-Z_1(u)y). \tag{4}$$

There is no interval (a, b) $([a] = [b], a \neq b)$ where we could establish the equality $\Phi(y) = \Phi_p(y)$ for $\forall y \in (a, b)$, because $\Phi_p(y)$ is an exponential function and $\Phi(y)$ is a polynomial with degree $[y/B_0(u)]$ in that interval. They could be equal at a countable number of points only.

Since one of the premises is incorrect, one can conclude that an exact solution of the linear integral one-dimensional equation of neutron slowing down has not been given in Perovich (1992), as declared.

References

Perovich S M 1992 *J. Phys. A: Math. Gen.* **25** 2969